

SISSA REF 129/99/EP
SWAT/243

Regular R–R and NS–NS BPS black holes

Matteo Bertolini^{a,1} and Mario Trigiante^b

^a*International School for Advanced Studies ISAS-SISSA and INFN
Sezione di Trieste, Via Beirut 2-4, 34013 Trieste, Italy*

^b*Department of Physics, University of Wales Swansea, Singleton Park
Swansea SA2 8PP, United Kingdom*

Abstract

We show in a precise group theoretical fashion how the generating solution of BPS regular black holes in $N = 8$ supergravity, which is known to be a solution also of a simpler $N = 2$ STU model truncation, can be characterized as purely NS–NS or R–R charged according to the way the corresponding STU model is embedded in the original $N = 8$ theory. In particular, one of these embeddings yields regular BPS black hole solutions carrying R–R charge whose microscopic description can be given in terms of bound states of D–branes only (“pure” R–R configurations). Within this embedding we will consider, as an explicit example, a four parameters solution for which we give both the full macroscopic and microscopic descriptions.

e-mail: teobert@sissa.it, m.trigiante@swansea.ac.uk

¹Address after November 1st: *NORDITA, Blegdamsvej 17, DK-2100 Copenhagen, Denmark.*
Supported in part by EEC under TMR contracts ERBFMRX–CT96–0045 and ERBFMRX–CT96–0012.

1 Introduction

After the discovery of D -branes [1], there have been successful microscopic computations for the entropy of some extremal and non-extremal black hole configurations which reproduced, at the microscopic level, the expected Beckenstein–Hawking behaviour [2]–[6]. However, despite these encouraging results, an open problem, nowadays, is still to find a general recipe to give this correspondence based on first principles other than specific computations. Actually, while gravity seems to describe the quantum properties of all black holes in a unified but incomplete way, string theory seems to give nice answers but loosing the unified character of the properties of different black holes. Actually it would be necessary to find a microscopic but still unified way of describing black hole physics in the context of string theory. In the last two years there have been in fact various attempts, especially within the AdS/CFT correspondence [7], to give an answer to this question relying on some unifying principles but a definite answer has not been found, yet. For recent progress in this direction see for example [8]–[14].

A complementary strategy, which could be helpful in this respect, is to take advantage of the U -duality properties of the black holes (like for instance the invariance of the entropy under U -duality transformations) and use them to infer the common underlined structure of very different black holes sharing the same entropy. In particular, if one is able to give a precise correspondence between at least one macroscopic solution (ultimately the 5 parameters generating one) and its microscopic description, and is *really* able to act on it via duality transformations in a precise and explicit way, one could derive the microscopic stringy description of any macroscopic solution. Even of those solutions (as the pure NS–NS ones) for which a microscopic entropy counting has not been achieved, yet. In our opinion, the possibility of having a control, both at macroscopic and microscopic level, on all regular black holes with a given entropy could shed further light on the very conceptual basis of the microscopic entropy within string theory. This is the spirit this paper relies on.

Some time ago it has been shown that the generating solution of regular $N = 8$ black holes can be characterized as a solution within the STU model [15]. Such a model is a $N = 2$ truncation of the $N = 8$ original theory [16]. Any regular BPS black hole solution within this model is a $1/2$ BPS soliton of the $N = 2$ theory but, from the original $N = 8$ point of view, it is in fact a $1/8$ supersymmetry preserving one (as any other regular BPS black hole solution should be: indeed the $1/2$ and $1/4$ solutions of $N = 8$ supergravity have vanishing entropy [17]). This important result enables one to concentrate on the rather simple structure of the STU model, then generating more general (and complicated)

solutions by U -duality transformations. Without specification of the proper embedding in the mother $N = 8$ theory these STU model solutions can be NS-NS, R-R or of a mixed nature. This distinction, from the 4 dimensional point of view, relies on the identification of the relevant (dimensionally reduced) 10 dimensional fields which enter dynamically in the solution. In particular, a *pure* R-R solution is one whose unique NS-NS field present is the metric tensor G_{MN} but all other fields are R-R. This case is particularly interesting, this being the setting where a macroscopic/microscopic correspondence is more suitable: indeed the microscopic interpretation of the solution can be given in terms of a bound state of D-branes without any NS-brane or KK states.

Actually, it is the algebraic characterization of scalars and vector fields which identifies the *microscopic* nature of a given solution. The aim of this paper is to define two different classes of embeddings (modulo S, T duality transformations) of the STU model within the $N = 8$ theory, in order for its solutions (and in particular the generating solution) to be NS-NS and R-R charged, respectively. In particular we shall consider a specific embedding yielding a pure R-R solution.

Although our main concern in this paper is the geometrical characterization of the STU models with different microscopic field content, in section 3 we will consider, as an explicit example, a four parameters solution of the STU model, which can be easily characterized, at the microscopic level, in terms of a bound state of $D4$ and $D0$ branes. The relation between macroscopic and microscopic parameters will be particularly simple and immediate.

2 The microscopic “nature” of the STU model

The 10 dimensional interpretation of the fields characterizing the solution depends on the embedding of the STU model inside the $N = 8$ theory. A powerful tool for a detailed study of these embeddings is based on the so-called *Solvable Lie Algebra* (SLA) approach. In the following we summarize the main features of this formalism while we refer to [18] for a complete review on the subject.

The solvable Lie algebra technique consists in defining a one to one correspondence between the scalar fields spanning a Riemannian homogeneous (symmetric) scalar manifold of the form $\mathcal{M} = G/H$ (G being a non-compact semisimple Lie group and H its maximal compact subgroup) and the generators of the solvable subalgebra $Solv$ of the isometry algebra \mathcal{G} defined by the well known Iwasawa decomposition:

$$\mathcal{G} = \mathcal{H} \oplus Solv \tag{2.1}$$

where \mathcal{H} is the compact algebra generating H . A Lie algebra G_s is *solvable* if for some integer $n \geq 1$, its n^{th} order *derived* algebra vanishes:

$$\begin{aligned}\mathcal{D}^{(n)}G_s &= 0 \quad \text{where} \\ \mathcal{D}G_s^{(1)} &\equiv [G_s, G_s] \quad ; \quad \mathcal{D}^{(k+1)}G_s \equiv [\mathcal{D}^{(k)}G_s, \mathcal{D}^{(k)}G_s]\end{aligned}$$

Since the 70-dimensional scalar manifold \mathcal{M}_{scal} of $N = 8$ supergravity has the above coset structure with $G = E_{7(7)}$ and $H = SU(8)$ it can be globally described as the group manifold generated by a solvable Lie algebra $Solv_7$, whose parameters are the scalar fields ϕ_i :

$$Solv_7 = \{T_i\} \quad \phi_i \leftrightarrow T_i \quad i = 1, \dots, 70$$

The solvable group generated by $Solv_7$ acts transitively on \mathcal{M}_{scal} . Considering the $N = 8$, $d = 4$ theory as the dimensional reduction on a torus T^6 of type IIA or IIB supergravity theories in $d = 10$, the solvable characterization of the NS-NS and R-R scalars in the four dimensional theory was worked out in [19, 20] and is achieved by decomposing the solvable algebra $Solv_7$ with respect to the solvable algebra $Solv_T + Solv_S$, where $Solv_T$ generates the moduli space of the torus $\mathcal{M}_T = SO(6, 6)/SO(6) \times SO(6)$ ($T = SO(6, 6)$ being the classical T -duality group), and $Solv_S$ generates the two dimensional manifold $SL(2, \mathbb{R})/SO(2)$ spanned by the dilaton ϕ and the axion $B_{\mu\nu}$ ($S = SL(2, \mathbb{R})$ being the S -duality group of the classical theory). Since in the formalism outlined above $Solv_T$ is naturally parameterized by the moduli scalars G_{ij} , B_{ij} (i, j denoting the directions inside the torus), and $Solv_S$ by ϕ and $B_{\mu\nu}$, the complement of $Solv_T + Solv_S$ inside $Solv_7$ is a nilpotent 32-dimensional subalgebra parameterized by the 32 R-R scalars. The general structure of the solvable algebra defined by the decomposition (2.1) is the direct sum of a subspace of the Cartan subalgebra CSA and the nilpotent space spanned by the *shift* operators corresponding to roots whose restriction to this Cartan subspace is positive:

$$Solv = \mathcal{C}_K \oplus \sum_{\alpha \in \Delta^+} \{E_\alpha\} \quad (2.2)$$

\mathcal{C}_K is the non-compact part of the CSA and Δ^+ is the space of those roots which are positive (non vanishing) with respect to \mathcal{C}_K .

In the case of the $N = 8$ theory in $d = 4$, $Solv$ is generated by the generators of the whole Cartan subalgebra of $\mathcal{E}_{7(7)}$ (the algebra generating the group $E_{7(7)}$, whose Cartan generators are non-compact) and all the shift operators corresponding to the positive roots of the same algebra. The Cartan generators correspond to the *radii* of the internal torus G_{ii} plus the dilaton ϕ , the positive roots correspond to the remaining T_6 moduli and enter

the structure of $Solv_T$ while the shift operators corresponding to the positive spinorial roots of the $SO(6,6)$ T -duality group are naturally parameterized by the R–R scalars. The precise correspondence between the positive roots of $E_{7(7)}$ and type IIA and type IIB fields is summarized in Table 2. Although this correspondence is fixed by the geometry, in what follows we shall define algebraically two different classes of *embeddings* of the STU model within the $N = 8$ theory which describe NS–NS or R–R charged solutions respectively. The embeddings within each class are related by an $S \times T$ conjugation which preserves, as a general property, the NS–NS and R–R nature of the fields .

Let us recall the main concepts on how to define the embedding of the STU from the reduction of the central charge matrix Z_{AB} of the $N = 8$ theory to its skew diagonal form, $Z^{\mathcal{N}}$. The latter procedure is a $SU(8)$ gauge fixing which can be defined by setting all off diagonal entries of Z_{AB} to zero, except the skew-diagonal ones, by means of a suitable 48-parameter $SU(8)$ transformation. The result is the central charge in its normal form:

$$Z^{\mathcal{N}} = \begin{pmatrix} Z_1 \epsilon & 0 & 0 & 0 \\ 0 & Z_2 \epsilon & 0 & 0 \\ 0 & 0 & Z_3 \epsilon & 0 \\ 0 & 0 & 0 & Z_4 \epsilon \end{pmatrix}, \quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (2.3)$$

This gauge fixing corresponds to a 48-parameter U -duality transformation on the 56 quantized charges $\vec{Q} = (p, q)$ and 54 scalar fields in the expression of the central charge such that the four skew-eigenvalues of the central charge will depend finally only on 8 quantized magnetic and electric charges $\vec{Q}^{\mathcal{N}} = (p^{\mathcal{N}}, q^{\mathcal{N}})$ (the *normal form* for the quantized charges) and on 6 scalar fields which define the vector and the scalar content of the STU model describing the generating solution. The scalar manifold of this $N = 2$ truncation is:

$$\mathcal{M}_{STU} = \left[\frac{SL(2, \mathbb{R})}{SO(2)} \right]^3 \quad (2.4)$$

The *centralizer* of $\vec{Q}^{\mathcal{N}}$, which is defined as the maximal subgroup G_C of $E_{7(7)}$ such that $G_C \cdot \vec{Q}^{\mathcal{N}} = \vec{Q}^{\mathcal{N}}$, is $SO(4,4)$ while the *centralizer* H_C of $Z^{\mathcal{N}}$ is $SO(4)^2$, maximal compact subgroup of G_C . On the other hand the *normalizer* $G_{\mathcal{N}}$ of $\vec{Q}^{\mathcal{N}}$, which is defined as the subgroup of $E_{7(7)}$ that commutes with the centralizer, $[G_{\mathcal{N}}, G_C] = 0$, is the isometry group of \mathcal{M}_{STU} , $[SL(2, \mathbb{R})]^3$, while its isotropy group $[SO(2)]^3$ is the *normalizer* $H_{\mathcal{N}}$ of $Z^{\mathcal{N}}$. Given the central charge in its normal form, $G_{\mathcal{N}}$ and G_C are then fixed within $E_{7(7)}$ and therefore also the embedding of the final STU model, \mathcal{M}_{STU} being given by $G_{\mathcal{N}}/H_{\mathcal{N}}$. The scalar content of the latter model, in terms of the $N = 8$ scalars, is defined by

embedding $Solv(\mathcal{M}_{STU})$ into $Solv_7$, \vec{Q}^N define the quantized charges of the model, while as usual the real and imaginary parts of the skew eigenvalues Z_k of the central charge define the physical dressed electric and magnetic charges of the interacting $N = 2$ model.

The above defined procedure of reduction of the central charge to its normal form, when applied to Z_{AB} in *different* bases, yields skew eigenvalues depending on the scalar and charge content of STU models embedded *differently* inside the original theory (the algebras \mathcal{G}_N and $\mathcal{G}_C \subset \mathcal{E}_{7(7)}$, generating G_N and G_C , would in general depend on the original basis of Z_{AB}). As we are going to explain in the following subsections there are essentially two physically different classes defined by this embedding.

2.1 The NS–NS STU model

Let us consider the central charge matrix in a basis $Z_{\hat{A}\hat{B}}$ in which the index \hat{A} of the **8** of $SU(8)$ splits in the following way: $\hat{A} = (a = 1, \dots, 4; a' = 1' \dots, 4')$, where a and a' index the $(\mathbf{4}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{4}')$ in the decomposition of the **8** with respect to $SU(4) \times SU(4)' = SU(8) \cap SO(6, 6)$ (this is the basis considered by Cvetič and Hull in defining their NS–NS 5–parameter solution, [21]). The group $SU(4) \times SU(4)'$ is the maximal compact subgroup of the classical T –duality group and decomposing with respect to it the **28** of $SU(8)$ will define which of the entries of $Z_{\hat{A}\hat{B}}$ correspond to R–R and which to NS–NS vectors (the former will transform in the spinorial of $SU(4)^2 \equiv SO(6)^2$):

$$\mathbf{28} \rightarrow (\mathbf{6}, \mathbf{1}') + (\mathbf{1}, \mathbf{6}') + (\mathbf{4}, \mathbf{4}') \quad (2.5)$$

the $(\mathbf{6}, \mathbf{1}') + (\mathbf{1}, \mathbf{6}')$ part consists of the two diagonal blocks Z_{ab} and $Z_{a'b'}$ and define the 12 NS–NS (complex) charges, while the spinorial $(\mathbf{4}, \mathbf{4}')$ correspond to the off–diagonal block $Z_{aa'}$ and define the 16 (complex) R–R charges. The skew–diagonal elements which will define Z_{NS}^N correspond then to NS–NS charges ($Z_{12}, Z_{34}, Z_{1'2'}, Z_{3'4'}$) and therefore the corresponding STU model will contain 4 NS–NS vector fields. Let us work out the embedding of G_N and G_C within $E_{7(7)}$. Let the simple roots of $\mathcal{E}_{7(7)}$ be α_n whose expression with respect to an orthonormal basis ϵ_n is the following:

$$\begin{aligned} \alpha_1 &= \epsilon_1 - \epsilon_2 ; \alpha_2 = \epsilon_2 - \epsilon_3 ; \alpha_3 = \epsilon_3 - \epsilon_4 \\ \alpha_4 &= \epsilon_4 - \epsilon_5 ; \alpha_5 = \epsilon_5 - \epsilon_6 ; \alpha_6 = \epsilon_5 + \epsilon_6 \\ \alpha_7 &= -\frac{1}{2}(\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 + \epsilon_5 + \epsilon_6) + \frac{\sqrt{2}}{2}\epsilon_7 \end{aligned} \quad (2.6)$$

The group $H_C = SO(4)^2 \subset SO(6)^2 \subset SO(6, 6)$ consists of four $SU(2)$ factors acting separately on the blocks $(1, 2)$, $(3, 4)$, $(1'2')$, $(3'4')$ of the central charge matrix. The centralizer at the level of quantized charges G_C on the other hand is the group $SO(4, 4)$ regularly

embedded in $SO(6, 6)$. If the latter is described by the simple roots $\alpha_1, \dots, \alpha_6$, a simple choice, modulo $S \times T$ conjugations, for the Dynkin diagram of \mathcal{G}_C would be $\alpha_3, \alpha_4, \alpha_5, \alpha_6$. The solvable subalgebra of \mathcal{G}_C consists of only NS–NS generators. The algebra \mathcal{G}_N , being characterized as the largest subalgebra of $Solv_7$ which commutes with \mathcal{G}_C , is immediately defined, modulo isomorphisms, to be the $[SL(2, \mathbb{R})]^3$ algebra corresponding to the roots $\beta_1 = \sqrt{2}\epsilon_7$, $\beta_2 = \epsilon_1 - \epsilon_2$ and $\beta_3 = \epsilon_1 + \epsilon_2$. The scalar manifold of the corresponding STU model has the form:

$$\mathcal{M}_{STU} = \frac{G_N}{H_N} = \frac{SU(1, 1)}{U(1)}(\beta_1) \times \frac{SO(2, 2)}{SO(2) \times SO(2)}(\beta_2, \beta_3) \quad (2.7)$$

The reason why the above expression has been written in a factorized form is to stress the different meaning of the two factors from the string point of view: the group $SU(1, 1)(\beta_1)$ represents the classical S –duality group of the theory and the corresponding factor of the manifold is parameterized by the dilaton ϕ and the axion $B_{\mu\nu}$. In the same way it can be shown that the second factor is parameterized by the scalars G_{44} , G_{55} , G_{45} and B_{45} and its isometry group acts as a classical T –duality, i.e. its restriction to the integers is the perturbative T –duality of string theory. This non–symmetric version of the STU model is the same as the one obtained as a consistent truncation of the toroidally compactified heterotic theory and therefore describes the generating solution also for this theory (the string interpretation of the 4 scalars spanning the second factor in \mathcal{M}_{STU} is in general non generalizable to the heterotic theory). Therefore its microscopic corresponding structure should be given in terms of NS states (fundamental string and NS5–brane states).

2.2 The R–R STU model

Let us start with the central charge matrix Z_{AB} obtained from $Z_{\hat{A}\hat{B}}$ through an orthogonal conjugation, such that the new index A of the **8** of $SU(8)$ assumes the values $A = 1, 1', 2, 2', \dots, 4, 4'$, the unprimed and primed indices spanning the **4** of the two $SU(4)$ subgroups previously defined. Let us now consider the decomposition of $SU(8)$ with respect to its subgroup $U(1) \times SU(2) \times SU(6)$ (which is the decomposition suggested by the Killing spinor analysis of the 1/8 BPS black holes) such that the **8** decomposes into a $(\mathbf{1}, \mathbf{2}, \mathbf{1})$ labeled by $i = 4, 4'$ and a $(\mathbf{1}, \mathbf{1}, \mathbf{6})$ labeled by $\tilde{i} = 1, 1', \dots, 3, 3'$. The **28** decomposes with respect to $U(1) \times SU(2) \times SU(6)$ in the following way:

$$\mathbf{28} \rightarrow (\mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{15}) + (\mathbf{1}, \mathbf{2}, \mathbf{6}) \quad (2.8)$$

where the singlet represents the diagonal block Z_{ij} , the $(\mathbf{1}, \mathbf{1}, \mathbf{15})$ the diagonal block $Z_{\tilde{i}, \tilde{j}}$ and the $(\mathbf{1}, \mathbf{2}, \mathbf{6})$ is spanned by the off diagonal entries $Z_{i, \tilde{j}}$. The skew–diagonal

entries which survive the gauge fixing procedure defined above and thus entering the new normal form of the central charge $Z_{RR}^{\mathcal{N}}$ are now $Z_1 = Z_{1,1'}$, $Z_2 = Z_{2,2'}$, $Z_3 = Z_{3,3'}$ and $Z_4 = Z_{4,4'}$, which are R–R charges. It is however interesting to notice that these four (complex) charges are part of the set of 10 R–R (complex) charges entering the diagonal blocks $(\mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{15})$. These charges can be immediately worked out by counting the entries with mixed primed and unprimed indices ($Z_{ab'}$) contained in these two blocks or in a group theoretical fashion by decomposing the $(\mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{15})$ in (2.8) and the $(\mathbf{4}, \mathbf{4}')$ in (2.5) with respect to a common subgroup $U(1) \times SU(3) \times SU(3)' = [U(1) \times SU(6)] \cap [SU(4) \times SU(4)']$. Both the decompositions contain a common representation $(\mathbf{1}, \mathbf{1}, \mathbf{1}') + (\mathbf{1}, \mathbf{3}, \mathbf{3}')$ describing 10 R–R central charges. The $\mathbf{3}$ and $\mathbf{3}'$ are spanned by the values 1, 2, 3 and $1', 2', 3'$ of the indices a and a' of the $\mathbf{4}$ and $\mathbf{4}'$ respectively. These charges correspond to the 1 + 9 vectors of an $N = 2$ truncation of the $N = 8$ theory with scalar manifold $SU(3, 3)/U(3) \times SU(3)$. A truncation of this theory yields the STU model defined by the new $SU(8)$ gauge fixing which brings the central charge Z_{AB} to the normal form $Z_{RR}^{\mathcal{N}}$. The 4 complex charges in $Z_{RR}^{\mathcal{N}}$ will depend on all the 8 R–R quantized magnetic and electric charges $\vec{Q}_{RR}^{\mathcal{N}}$ and the 6 scalar fields of the new STU model. Therefore, differently to the previous defined class, in this case all gauge fields (and hence the corresponding charges) come from R–R 10 dimensional forms. The centralizer $SO(4, 4)$ of $\vec{Q}_{RR}^{\mathcal{N}}$ is now no more contained inside $SO(6, 6)$ and therefore its solvable algebra contains R–R generators as well. As a common feature of the truncations belonging to this class, the scalars entering each quaternionic multiplet split into 2 NS–NS and 2 R–R. Indeed the centralizer $SO(4, 4)$ is now the isometry group of the manifold $SO(4, 4)/SO(4) \times SO(4)$ describing 16 hyperscalars and therefore its solvable algebra has 8 R–R and 8 NS–NS generators ².

In order to specify a particular truncation within the class one should define the simple roots of $SO(4, 4)$ and of the isometry group $[SL(2, \mathbb{R})]^3$ of the STU model, which in turn determines $Solv(\mathcal{M}_{STU})$ and thus the scalar content of the model. An interesting possibility is the one where the system of simple roots for $SO(4, 4)$ is chosen to be:

$$\begin{aligned}\gamma_1 &= \epsilon_1 + \epsilon_2 ; \gamma_3 = \epsilon_3 + \epsilon_4 ; \gamma_4 = \epsilon_5 + \epsilon_6 \\ \gamma_2 &= \alpha_7\end{aligned}\tag{2.9}$$

Here the root $\beta_1 = \sqrt{2}\epsilon_7 = \sum_{i=1}^4 \gamma_i$ belongs to the $SO(4, 4)$ root space. In the solvable language, since the Cartan generator and the shift operator corresponding to this root

²Notice that any solution within a Calabi–Yau compactification of type II string lies in this class. Indeed all the vector fields surviving the compactification come from R–R forms, both for type IIA and type IIB theories.

are parameterized by ϕ and $B_{\mu\nu}$, these two scalars are now part of a hypermultiplet, known as the *universal sector*. The isometry group of the STU model which commutes with the above defined $SO(4,4)$ centralizer is generated by a $[SL(2, \mathbb{R})]^3$ algebra which is regularly embedded in the isometry group $GL(6, \mathbb{R})$ of the classical moduli space of T^6 and defined by the following roots:

$$\beta_1 = \epsilon_1 - \epsilon_2 ; \beta_2 = \epsilon_3 - \epsilon_4 ; \beta_3 = \epsilon_5 - \epsilon_6 \quad (2.10)$$

The scalar manifold of this STU model is now symmetric among S, T, U since it is contained in the moduli space of T^6 (its scalars are all NS–NS but there is no ϕ and $B_{\mu\nu}$):

$$\mathcal{M}_{STU} = \frac{G_N}{H_N} = \frac{SU(1,1)}{U(1)}(\beta_1) \times \frac{SU(1,1)}{U(1)}(\beta_2) \times \frac{SU(1,1)}{U(1)}(\beta_3) \quad (2.11)$$

From Table 2 we can read out the scalar content of this model: G_{45} , G_{57} , G_{89} and 3 radii (the latter being Cartan generators). The interest in the above embedding is that all the excited scalar fields, although NS–NS, come from the metric tensor G_{MN} rather than from the antisymmetric tensor B_{MN} (on the contrary, and this is a common feature of all embeddings falling in this class, all charges are R–R). This means that this particular embedding represents a *pure* R–R solution whose microscopic description can be easily given in terms of D–branes only. Therefore, it is likely to look for the generating solution within this particular embedding because this is the case where a microscopic entropy counting can be more easily performed.

Considering the embedding defined by eq.s (2.9)-(2.11), in the framework of type IIA supergravity the ten dimensional fields which contribute to the 4 dimensional solution are G_{MN} , A_M , A_{MNP} while B_{MN} can be consistently put to zero. Therefore one can easily see that the microscopic configuration corresponding to a regular solution (either generating or not) within this pure R–R embedding should be given in terms of a 1/8 supersymmetry preserving bound state of $D0$, $D2$, $D4$ and $D6$ branes without the presence of any KK or NS–brane state. In the next section we will come back on this issue by considering an explicit R–R solution of the STU model, giving both its macroscopic and microscopic description.

3 A pure R–R solution and its microscopic description

Let us now consider a specific example, namely a four parameter solution within the STU model for which we shall give a microscopic description. From the macroscopic point of view this solution is analogous to the one described in [22]. Other macroscopic solutions of the STU model have been obtained, for instance, in [23, 24, 25, 26].

Let us very briefly remind the structure of the STU model while a complete treatment has been carried out in [25, 26]. The STU model is characterized by a $N = 2$ supergravity theory coupled to 3 vector multiplets whose scalars spans the manifold \mathcal{M}_{STU} , eq.(2.4). The total number of scalar fields in the game is 6 ($z_i = a_i + ib_i$, $i = 1, 2, 3$) while the number of charges (p^Λ, q_Λ) is 8 (4 electric and 4 magnetic). In the framework of the STU model, the local realization on moduli space \mathcal{M}_{STU} of the $N = 2$ supersymmetry algebra central charge Z and of the 3 *matter* central charges Z^i associated with the 3 matter vector fields are related to the $N = 8$ central charges eigenvalues in the following way:

$$Z = i Z_4 \quad , \quad Z^i = h^{ij*} \partial_{j*} Z = \mathbb{P}_\alpha^i Z^\alpha \quad (3.1)$$

where $\mathbb{P}_\alpha^i = 2b_i(r)$ is the vielbein transforming rigid indices α (the one characterizing the eigenvalues of the $N = 8$ central charge in its normal form, eq.(2.3)) to curved indices i (see [25], section 3, for details).

The killing spinor equations characterizing the BPS black hole solution translate into first order differential equations for the relevant bosonic fields once suitable ansätze are adopted. As a standard procedure, the vanishing of the gravitino transformation rule along killing spinor directions implies a condition for the metric while the vanishing of the dilatino transformations rule translates into equations for the scalar fields. The ansätze for the metric $G_{\mu\nu}$ and the complex scalars z^i are the following:

$$\begin{aligned} ds^2 &= e^{2\mathcal{U}(r)} dt^2 - e^{-2\mathcal{U}(r)} d\vec{x}^2 & (r^2 = \vec{x}^2) \\ z^i &\equiv z^i(r) \end{aligned} \quad (3.2)$$

After some algebra one can see that, in the case in which the central charge is taken to be real, the structure of the first order BPS equations turns out to be the following:

$$\begin{aligned} \frac{dz^i}{dr} &= \mp 2 \left(\frac{e^{\mathcal{U}(r)}}{r^2} \right) h^{ij*} \partial_{j*} |Z(z, \bar{z}, p, q)| \\ \frac{d\mathcal{U}}{dr} &= \mp \left(\frac{e^{\mathcal{U}(r)}}{r^2} \right) |Z(z, \bar{z}, p, q)| \end{aligned} \quad (3.3)$$

which is a system of first order differential equations. Working out the underlined geometric structure of the STU model, the above system of equations can be made explicit in terms of the scalar fields and the quantized charges (p^Λ, q_Λ) characterizing the model. This has been fully worked out in [25, 26] (where the same conventions and notations has been used), see in particular the appendices for explicit formulæ.

In this section we shall focus a particular (4-parameter) regular solution for which the microscopic description turns out to be particularly nice. On this solution the central

charge eigenvalues Z_α, Z_4 are pure imaginary. This condition fixes not only the $[SO(2)]^3$ symmetry of the model but also the overall phase θ of the four central charge eigenvalues ($\theta = 0 \bmod 2\pi$), yielding just four independent invariants $|Z_\alpha|, |Z_4|$.

Since the central charge Z_4 is set to be imaginary (i.e. Z real), the system of eqs. (3.3) may be rewritten in the simpler form:

$$\begin{aligned}\frac{dz^i}{dr} &= \mp \left(\frac{e^{\mathcal{U}(r)}}{r^2} \right) h^{ij*} \nabla_{j*} Z(z, \bar{z}, p, q) = \mp \left(\frac{e^{\mathcal{U}(r)}}{r^2} \right) Z^i(z, \bar{z}, p, q) \\ \frac{d\mathcal{U}}{dr} &= \mp \left(\frac{e^{\mathcal{U}(r)}}{r^2} \right) Z(z, \bar{z}, p, q)\end{aligned}\quad (3.4)$$

It is possible moreover to show that the reality of Z is consistent with the regularity of the solution provided we set $p^0 = 0$. The conditions $Z_\alpha = -\bar{Z}_\alpha$ (and therefore $Z^i = -\bar{Z}^i$) imply that the three axions are double-fixed: $a_{1,2,3}(r) \equiv a_{1,2,3}^f$. They require also that three electric quantized charges vanish, namely: $q_1 = q_2 = q_3 = 0$.

Hence the quantized charges left are (q_0, p^1, p^2, p^3) and the system of first order differential equations our solution has to fulfill reduces considerably. Indeed the equations for the dilatons and for \mathcal{U} decouple from the axions and may be solved independently:

$$\begin{aligned}\frac{db_1}{dr} &= \pm \left(\frac{e^{\mathcal{U}}}{r^2} \right) \sqrt{-\frac{b_1}{2b_2b_3}} (p^1 b_2 b_3 - p^2 b_1 b_3 - p^3 b_1 b_2 + q_0) \\ \frac{db_2}{dr} &= \pm \left(\frac{e^{\mathcal{U}}}{r^2} \right) \sqrt{-\frac{b_2}{2b_1b_3}} (-p^1 b_2 b_3 + p^2 b_1 b_3 - p^3 b_1 b_2 + q_0) \\ \frac{db_3}{dr} &= \pm \left(\frac{e^{\mathcal{U}}}{r^2} \right) \sqrt{-\frac{b_3}{2b_1b_2}} (-p^1 b_2 b_3 - p^2 b_1 b_3 + p^3 b_1 b_2 + q_0) \\ \frac{d\mathcal{U}}{dr} &= \pm \left(\frac{e^{\mathcal{U}}}{r^2} \right) \frac{1}{2\sqrt{2}\sqrt{-b_1b_2b_3}} (p^1 b_2 b_3 + p^2 b_1 b_3 + p^3 b_1 b_2 + q_0)\end{aligned}\quad (3.5)$$

The 3 equations for the axions and the one on the reality of the central charge give the 4 r -independent relations our solution should fulfill:

$$\begin{aligned}\frac{da_1}{dr} &= 0 = -(a_3 b_2 + a_2 b_3) p^1 + (a_3 b_1 - a_1 b_3) p^2 + (a_2 b_1 - a_1 b_2) p^3 \\ \frac{da_2}{dr} &= 0 = -(a_3 b_1 + a_1 b_3) p^2 + (a_3 b_2 - a_2 b_3) p^1 + (a_1 b_2 - a_2 b_1) p^3 \\ \frac{da_3}{dr} &= 0 = -(a_1 b_2 + a_2 b_1) p^3 + (a_1 b_3 - a_3 b_1) p^2 + (a_2 b_3 - a_3 b_2) p^1 \\ Im Z &= 0 = (a_3 b_2 + a_2 b_3) p^1 + (a_3 b_1 + a_1 b_3) p^2 + (a_2 b_1 + a_1 b_2) p^3\end{aligned}\quad (3.6)$$

The fixed values for the scalar fields (namely the values the scalars get at the horizon,

[27]) are:

$$\begin{aligned} b_1^{fix} &= -\sqrt{\frac{q_0 p^1}{p^2 p^3}} \ , \ b_2^{fix} = -\sqrt{\frac{q_0 p^2}{p^1 p^3}} \ , \ b_3^{fix} = -\sqrt{\frac{q_0 p^3}{p^1 p^2}} \\ a_1^{fix} &= 0 \ , \ a_2^{fix} = 0 \ , \ a_3^{fix} = 0 \end{aligned} \quad (3.7)$$

Let us introduce the four harmonic functions as follows:

$$\begin{aligned} H_\alpha(r) &= A_\alpha + k_\alpha/r \ (\alpha = 0, 1, 2, 3) \\ k_0 &= \sqrt{2} q_0 \ , \ k_i = \sqrt{2} p^i \end{aligned} \quad (3.8)$$

It is easy to see that performing the following ansatz for the b_i and the scalar function \mathcal{U} :

$$b_1 = -\sqrt{\frac{H_0 H_1}{H_2 H_3}} \ , \ b_2 = -\sqrt{\frac{H_0 H_2}{H_1 H_3}} \ , \ b_3 = -\sqrt{\frac{H_0 H_3}{H_1 H_2}} \ , \ \mathcal{U} = -\frac{1}{4} \ln(H_0 H_1 H_2 H_3) \quad (3.9)$$

both the first and second order differential equations are satisfied. Choosing the metric to be asymptotically flat and standard values for the dilatons at infinity, the four constants A_α are set to be all equal to 1. The solution, consisting of the three b_i , the double-fixed a_i and \mathcal{U} is expressed in terms of 4 independent charges (and four harmonic functions): q_0, p^1, p^2, p^3 . According to the ansatz (3.2) the metric has the following form:

$$ds^2 = (H_0 H_1 H_2 H_3)^{-1/2} dt^2 - (H_0 H_1 H_2 H_3)^{1/2} d\vec{x}^2 \quad (3.10)$$

and the macroscopic entropy, according to Beckenstein–Hawking formula, reads:

$$S_{macro} = 2\pi \sqrt{q_0 p^1 p^2 p^3} \quad (3.11)$$

As far as the vector fields are concerned, their form in terms of the harmonic functions introduced above is analogous to the one in the solution of [22] and we shall not give it here. Let us now move to the microscopic description of the above solution. This will be easily obtained starting from the explicit expression of the $N = 8$ central charge eigenvalues. Comparing eq.s (3.3) and (3.5) one can see that the central charge eigenvalues at spatial infinity, when written in terms of the quantized charges, reads (in rigid indices):

$$\begin{aligned} Z_1 &= \frac{i}{2\sqrt{2}}(q_0 - p^2 - p^3 + p^1) \\ Z_2 &= \frac{i}{2\sqrt{2}}(q_0 + p^2 - p^3 - p^1) \\ Z_3 &= \frac{i}{2\sqrt{2}}(q_0 - p^2 + p^3 - p^1) \\ Z_4 &= \frac{i}{2\sqrt{2}}(q_0 + p^2 + p^3 + p^1) \end{aligned} \quad (3.12)$$

According to the previous discussion, if the STU model is embedded in the full $N = 8$ theory according to formulæ(2.8),(2.9) and (2.10), the microscopic description of the above solution can be given in terms of the intersection of four bunches of D-branes. Indeed, as explained in the previous section, provided a pure R–R embedding, the central charge Z_4 (which represents the $N = 2$ graviphoton dressed charge) and the matter charges Z_α ($\alpha = 1, 2, 3$) are related to the gauge fields coming from the 10 dimensional R–R 3–form A_{MNP} coupling to D2 and D4–branes and from the R–R 1–form A_M coupling to D0 and D6–branes. Our solution is hence described, at the microscopic level, as a 1/8 supersymmetry preserving intersection of 4 bunches of these D–branes. The fact that each of the central charge eigenvalues is real or pure imaginary (in our case they are all pure imaginary) implies that the solution is pure electric or magnetic, that is it is not made of electromagnetic dual objects.

One can think, for instance, of 3 bunches of orthogonal $D4$ branes (N_1, N_2, N_3 , respectively) wrapped on the internal torus T^6 with N_0 $D0$ branes on top of them. Let us consider the torus T^6 to be labeled with coordinates x^4, x^5, \dots, x^9 while the 4 dimensional space–time with coordinates x^0, x^1, x^2, x^3 . The $D4$ –branes are positioned in the following way:

	x^4	x^5	x^6	x^7	x^8	x^9
N_1	\cdot	\cdot	\times	\times	\times	\times
N_2	\times	\times	\cdot	\cdot	\times	\times
N_3	\times	\times	\times	\times	\cdot	\cdot

Table 1: The position of the $D4$ branes on the compactifying torus: for any given brane the directions labeled with \times are Neumann while those labeled with \cdot are Dirichlet.

The above configuration is 1/8 supersymmetric and adding any number of $D0$ branes the number of preserved supersymmetries does not change, [28]. The precise relation between the above microscopic configuration and the macroscopic solution can be more easily derived by writing the expression of the $E_{7(7)}$ quartic invariant as in [29]:

$$\begin{aligned}
J_4 = & (|Z_1| + |Z_2| + |Z_3| + |Z_4|) (|Z_1| - |Z_2| - |Z_3| + |Z_4|) (-|Z_1| + |Z_2| - |Z_3| + |Z_4|) \\
& (-|Z_1| - |Z_2| + |Z_3| + |Z_4|) + 8|Z_1||Z_2||Z_3||Z_4| (\cos \theta - 1)
\end{aligned} \tag{3.13}$$

where, as well known, the entropy of the solution is $S = \pi\sqrt{J_4}$. In the case at hand $\theta = 0 \bmod 2\pi$ and the last term in the above equation drops out (according to the fact that it is a four, rather than a five parameters solution). The above expression reduces

to:

$$J_4 = s_0 s_1 s_2 s_3 \quad (3.14)$$

where, using relations (3.12), it follows:

$$\begin{aligned} s_0 &\equiv (|Z_1| + |Z_2| + |Z_3| + |Z_4|) = \sqrt{2}q_0 \\ s_1 &\equiv (|Z_1| - |Z_2| - |Z_3| + |Z_4|) = \sqrt{2}p^1 \\ s_2 &\equiv (-|Z_1| + |Z_2| - |Z_3| + |Z_4|) = \sqrt{2}p^2 \\ s_3 &\equiv (-|Z_1| - |Z_2| + |Z_3| + |Z_4|) = \sqrt{2}p^3 \end{aligned} \quad (3.15)$$

As noticed in [29], the charge vector basis we have chosen turns out to be the suitable one for the microscopic identification, as for reading off the values of the integers N_α from the relations (3.12). First notice that the 4 dimensional charge of a wrapped D p -brane is $Q_p = \hat{\mu}_p \cdot V_p / \sqrt{V_6}$ where $\hat{\mu}_p = \sqrt{2\pi}(2\pi\sqrt{\alpha'})^{3-p}$ is the normalized D p -brane charge density in ten dimensions. Provided the asymptotic values of the dilatons, which parameterize the radii of the compactifying torus and which has been taken to be unitary, it turns out that, in units where $\alpha' = 1$, the four dimensional quanta of charge for *any* kind of (wrapped) D p -brane is equal to $\sqrt{2\pi}$. On the contrary, our quantized charges (p^Λ, q_Λ) are integer valued. Hence the entropy formula (3.11) is reproduced microscopically by the above D-branes configuration, table 1, if we have precisely $N_0 = q_0$, $N_1 = p^1$, $N_2 = p^2$, $N_3 = p^3$. Indeed the microscopic entropy counting for this configuration has been performed in [30] and gives:

$$S_{micro} = 2\pi\sqrt{N_0 N_1 N_2 N_3} \quad (3.16)$$

which exactly matches expression (3.11). From the configuration in table 1 one can obtain, by various dualities, other four parameters solutions. For instance, T -dualizing on the whole T^6 , one obtains a configuration made of N_0 D6-branes and 3 bunches of (N_1, N_2, N_3) D2-branes localized in the planes (x^4, x^5) , (x^6, x^7) , (x^8, x^9) respectively. Under more general T-dualities (i.e. those corresponding to $SO(6, 6; \mathbb{Z})$ elements which have the effect of “tilted” T-dualities, from the microscopic point of view) one can also get D-brane configurations with the NS anti-symmetric tensor switched on. Or even, acting with the full U -duality group, configurations with other stringy states present. In any case, this being not the generating solution, one cannot recover the full U -duality spectrum.

From the above four parameters configuration one could infer, in fact, the microscopic structure of the *five* parameters one. In [31] it has been noticed that the 5 parameters

solution could be obtained from the above switching on a EM flux F on the D4-branes world-volume in such a way to preserve supersymmetry. This would imply effective additional D2 and D0 charge [32] and, from a macroscopic point of view, the switching on of the real parts of the three matter central charges Z_1, Z_2, Z_3 (which would represent effective D2-brane charge). The microscopic entropy counting, in that case, should be better performed in a T -dual picture. Indeed, T -dualizing along x^5, x^7, x^9 one would end up with four bunches of type IIB $D3$ branes (N_0, N_1, N_2, N_3 respectively) at angles. The overall angle (the fifth parameter) would be determined essentially by the flux F and would be the right one in order to preserve supersymmetry, that is, an $U(3)$ angle, [28]. For $F = 0$ one would get the D3-branes to be orthogonal, hence recovering the four parameter solution [33] (although in the T -dual, type IIB, picture).

4 Comments and Conclusions

The main aim of the present article was to define in a precise mathematical fashion a connection between the *macroscopic* analysis of 1/8 BPS black hole solutions of $N = 8, d = 4$ supergravity carried out in [15, 17, 25, 26] and the *microscopic* description of the subclass of these solutions carrying R-R charge in terms of D-branes. To this end it was necessary to single out in the U -duality orbit of 1/8 BPS black holes those charged with respect to R-R vector fields, once the $N = 8$ theory is interpreted as the low-energy limit of type II superstring on T^6 . The first step in this direction was to characterize geometrically the embedding of a class of STU models describing R-R charged 1/8 BPS black holes within the $d = 4$ maximal supergravity. To achieve this we used the techniques developed in [19, 20] in order to characterize geometrically the R-R and the NS-NS ten dimensional origin of the fields in the $N = 8, d = 4$ theory. As a byproduct we defined a *dual* class of STU models describing NS-NS charged solutions in the same mathematical fashion and the U -duality relation between the two classes of STU models can be inferred from their embedding in the larger $N = 8$ theory (this transformation is in the group $U = E_{7(7)}$ but not in the subgroup $S \times T = SL(2, \mathbb{R}) \times SO(6, 6)$, since it does not preserve the R-R and NS-NS identities of the fields, while the models within each class are related by S and T dualities). Eventually we focused on a particular representative of the R-R class of STU models and interpreted its fields in terms of their ten dimensional origin. A particular 4-parameter solution of this model was considered and a microscopic interpretation of it was given in terms D-branes (a configuration of $D4$ and $D0$ -branes, in the framework of type IIA theory).

All these efforts have in fact some relevance in the study of regular BPS black hole

solutions of toroidally compactified type II string (or M) theory. On the one hand the geometric characterization given allows one to identify the original ten dimensional fields entering a given black hole solution quite easily and hence to infer its microscopic structure. On the other hand, once a solution is given then one can find other solutions via U -duality transformations. The idea underlying this kind of reasoning is to try to have a precise control on both the macroscopic and microscopic structures of all regular stringy black holes related by U -duality transformations, hence sharing the same entropy. Starting from a configuration for which a microscopic entropy counting is known (as for instance pure D-branes configurations) one can then have an entropy prediction and a description, both at macroscopic and microscopic levels, of those configurations for which a microscopic entropy counting is out of present reach. This could help in revealing the underlined common properties of very different black holes sharing the same entropy and hence giving some insights into the basic properties of stringy oriented microscopic entropy counting.

To accomplish such a program and then recover the full 56 dimensional U -duality orbit it is necessary to consider the five parameter generating solution, which would be, as already pointed out, intrinsically dyonic. Although there exist in the literature some 5 parameters generating solutions, [34, 35, 26], the interpretation, at the microscopic level, of the parameters entering these solution is quite difficult, especially as far as the fifth one is concerned. Hence a simple and clear description of the generating solution, both at macroscopic and microscopic level, is still missing. The completion of this program is left to a future work.

Acknowledgments

We would like to thank V. Balasubramanian and F. Larsen for very useful discussions and the organizers of the ICTP conference on “Black holes Physics” during which part of this work was done. We also acknowledge useful discussions with A. Hammou, F. Morales, C.A. Scrucca, R. Russo, M. Serone, G. Bonelli, L. Andrianopoli, R. D’Auria and S. Ferrara. Finally, we would like to thank P. Frè for reading a preliminary version of the manuscript. One of us, M.T., would like to thank SISSA for the kind hospitality.

References

- [1] J. Polchinski, Phys. Rev. Lett. **75** (1995) 4724.
- [2] A. Strominger and C. Vafa, Phys. Lett. **B379** (1996) 99.
- [3] J. Maldacena and A. Strominger, Phys. Rev. Lett. **77** (1996) 428.

- [4] C.V. Johnson, R.R. Khuri and R.C. Myers, Phys. Lett. **B378** (1996) 78.
- [5] D. Kaplan, D. Lowe, J. Maldacena and A. Strominger, Phys. Rev. **D55** (1997) 4898.
- [6] J. Maldacena, A. Strominger and E. Witten, JHEP **12** (1997) 2.
- [7] J. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231.
- [8] A. Strominger, JHEP **02** (1998) 009.
- [9] P. Claus, M. Derix, R. Kallosh, J. Kumar, P. K. Townsend and A. Van Proeyen, Phys. Rev. Lett. **81** (1998) 4553.
- [10] J.R. David, G. Mandal and S.R. Wadia, Nucl. Phys. **B544** (1999) 590.
- [11] G.W. Gibbons and P.K. Townsend, Phys. Lett. **B454** (1999) 187.
- [12] S. Carlip, Phys. Rev. Lett. **82** (1999) 2828.
- [13] G.L. Cardoso, B. de Wit and T. Mohaupt, “*Deviations from the Area law for super-symmetric black holes*”, hep-th/9904005.
- [14] F. Larsen and E. Martinec, “*Currents and Moduli in the $(4,0)$ theory*”, hep-th/9909088.
- [15] L. Andrianopoli, R. D’Auria, S. Ferrara, P. Frè and M. Trigiante, Nucl. Phys. **B509** (1998) 463.
- [16] E. Cremmer and B. Julia, Phys. Lett. **B76** (1978) 409.
- [17] G. Arcioni, A. Ceresole, F. Cordaro, R. D’Auria, P. Frè, L. Gualtieri and M. Trigiante, Nucl. Phys. **B542** (1999) 273.
- [18] D.V. Alekseevskii, Math. USSR Izvestija, **Vol. 9** (1975), No.2; for a review see M. Trigiante, “*Dualities in Supergravity and Solvable Lie Algebras*”, hep-th/9801144, PhD thesis.
- [19] L. Andrianopoli, R. D’Auria, S. Ferrara, P. Frè, R. Minasian and M. Trigiante, Nucl. Phys. **B493** (1997) 249.
- [20] L. Andrianopoli, R. D’Auria, S. Ferrara, P. Frè and M. Trigiante, Nucl. Phys. **B496** (1997) 617.
- [21] M. Cvetič and C. M. Hull, Nucl. Phys. **B480** (1996) 296.

- [22] S. Ferrara and R. Kallosh, Phys. Rev. **D54** (1996) 1514.
- [23] M.J. Duff, James T. Liu and J. Rahmfeld, Nucl. Phys. **B459** (1996) 125; K. Behrndt, R. Kallosh, J. Rahmfeld, M. Shmakova and W.K. Wong, Phys. Rev. **D54** (1996) 6293.
- [24] K. Behrndt, G.L. Cardoso, B. de Wit, D. Lust, T. Mohaupt and W. A. Sabra, Phys. Lett. **B429** (1998) 289.
- [25] M. Bertolini, P. Frè and M. Trigiante, Class. Quant. Grav. **16** (1999) 1519.
- [26] M. Bertolini, P. Frè and M. Trigiante, Class. Quant. Grav. **16** (1999) 2987.
- [27] S. Ferrara, R. Kallosh and A. Strominger, Phys. Rev. **D52** (1995) 5412; A. Strominger, Phys. Lett. **B383** (1996) 39; S. Ferrara and R. Kallosh, Phys. Rev. **D54** (1996) 1525.
- [28] P. K. Townsend Nucl. Phys. Proc. Suppl. **67** (1998) 88; M. M. Sheikh-Jabbari, Phys. Lett. **B420** (1998) 279.
- [29] S. Ferrara and J. Maldacena, Class. Quant. Grav. **15** (1998) 749.
- [30] V. Balasubramanian and F. Larsen, Nucl. Phys. **B478** (1996) 199; V. Balasubramanian, F. Larsen and R. Leigh, Phys. Rev. **D 57** (1998) 3509.
- [31] V. Balasubramanian, “*How to Count the States of Extremal Black Holes in $N = 8$ Supergravity*”, hep-th/9712215.
- [32] M. Douglas, “*Branes within Branes*”, hep-th/9512077; M. Green, J.A. Harvey and G. Moore, Class. Quant. Grav. **14** (1997) 47.
- [33] K. Behrndt, D. Lust, W.A. Sabra, Phys. Lett. **B418** (1998) 303.
- [34] M. Cvetič, A. Tseytlin, Phys. Rev. **D53** (1996) 5619; Erratum-ibid. **D55** (1997) 3907.
- [35] M. Cvetič and D. Youm, Nucl. Phys. **B472** (1996) 249.

IIA	IIB	$\alpha_{m,n}$	ϵ_i -components	α_i -components
A_9	ρ	$\alpha_{1,1}$	$\frac{1}{2}(-1, -1, -1, -1, -1, -1, \sqrt{2})$	$(0, 0, 0, 0, 0, 0, 1)$
$B_{8,9}$	$B_{8,9}$	$\alpha_{2,1}$	$(0, 0, 0, 0, 1, 1, 0)$	$(0, 0, 0, 0, 0, 1, 0)$
$G_{8,9}$	$G_{8,9}$	$\alpha_{2,2}$	$(0, 0, 0, 0, 1, -1, 0)$	$(0, 0, 0, 0, 1, 0, 0)$
A_8	$A_{8,9}$	$\alpha_{2,3}$	$\frac{1}{2}(-1, -1, -1, -1, 1, 1, \sqrt{2})$	$(0, 0, 0, 0, 0, 1, 1)$
$B_{7,8}$	$B_{7,8}$	$\alpha_{3,1}$	$(0, 0, 0, 1, 1, 0, 0)$	$(0, 0, 0, 1, 1, 1, 0)$
$G_{7,8}$	$G_{7,8}$	$\alpha_{3,2}$	$(0, 0, 0, 1, -1, 0, 0)$	$(0, 0, 0, 1, 0, 0, 0)$
$B_{7,9}$	$B_{7,9}$	$\alpha_{3,3}$	$(0, 0, 0, 1, 0, 1, 0)$	$(0, 0, 0, 1, 0, 1, 0)$
$G_{7,9}$	$G_{7,9}$	$\alpha_{3,4}$	$(0, 0, 0, 1, 0, -1, 0)$	$(0, 0, 0, 1, 1, 0, 0)$
$A_{7,8,9}$	$A_{7,8}$	$\alpha_{3,4}$	$\frac{1}{2}(-1, -1, -1, 1, 1, -1, \sqrt{2})$	$(0, 0, 0, 1, 1, 1, 1)$
A_7	$A_{7,9}$	$\alpha_{3,6}$	$\frac{1}{2}(-1, -1, -1, 1, -1, 1, \sqrt{2})$	$(0, 0, 0, 1, 0, 1, 1)$
$B_{6,7}$	$B_{6,7}$	$\alpha_{4,1}$	$(0, 0, 1, 1, 0, 0, 0)$	$(0, 0, 1, 2, 1, 1, 0)$
$G_{6,7}$	$G_{6,7}$	$\alpha_{4,2}$	$(0, 0, 1, -1, 0, 0, 0)$	$(0, 0, 1, 0, 0, 0, 0)$
$B_{6,8}$	$B_{6,8}$	$\alpha_{4,3}$	$(0, 0, 1, 0, 1, 0, 0)$	$(0, 0, 1, 1, 1, 1, 0)$
$G_{6,8}$	$G_{6,8}$	$\alpha_{4,4}$	$(0, 0, 1, 0, -1, 0, 0)$	$(0, 0, 1, 1, 0, 0, 0)$
$B_{6,9}$	$B_{6,9}$	$\alpha_{4,4}$	$(0, 0, 1, 0, 0, 1, 0)$	$(0, 0, 1, 1, 0, 1, 0)$
$G_{6,9}$	$G_{6,9}$	$\alpha_{4,6}$	$(0, 0, 1, 0, 0, -1, 0)$	$(0, 0, 1, 1, 1, 0, 0)$
$A_{6,7,9}$	$A_{6,7}$	$\alpha_{4,7}$	$\frac{1}{2}(-1, -1, 1, 1, -1, -1, \sqrt{2})$	$(0, 0, 1, 2, 1, 1, 1)$
$A_{6,8,9}$	$A_{6,8}$	$\alpha_{4,8}$	$\frac{1}{2}(-1, -1, 1, -1, 1, -1, \sqrt{2})$	$(0, 0, 1, 1, 1, 1, 1)$
A_6	$A_{6,9}$	$\alpha_{4,9}$	$\frac{1}{2}(-1, -1, 1, -1, -1, 1, \sqrt{2})$	$(0, 0, 1, 1, 0, 1, 1)$
$A_{6,7,8}$	$A_{6,7,8,9}$	$\alpha_{4,10}$	$\frac{1}{2}(-1, -1, 1, 1, 1, 1, \sqrt{2})$	$(0, 0, 1, 2, 1, 2, 1)$
$B_{5,6}$	$B_{5,6}$	$\alpha_{5,1}$	$(0, 1, 1, 0, 0, 0, 0)$	$(0, 1, 2, 2, 1, 1, 0)$
$G_{5,6}$	$G_{5,6}$	$\alpha_{5,2}$	$(0, 1, -1, 0, 0, 0, 0)$	$(0, 1, 0, 0, 0, 0, 0)$
$B_{5,7}$	$B_{5,7}$	$\alpha_{5,3}$	$(0, 1, 0, 1, 0, 0, 0)$	$(0, 1, 1, 2, 1, 1, 0)$
$G_{5,7}$	$G_{5,7}$	$\alpha_{5,4}$	$(0, 1, 0, -1, 0, 0, 0)$	$(0, 0, 1, 0, 0, 0, 0)$
$B_{5,8}$	$B_{5,8}$	$\alpha_{5,5}$	$(0, 1, 0, 0, 1, 0, 0)$	$(0, 1, 1, 1, 1, 1, 0)$
$G_{5,8}$	$G_{5,8}$	$\alpha_{5,6}$	$(0, 1, 0, 0, -1, 0, 0)$	$(0, 0, 0, 1, 0, 0, 0)$
$B_{5,9}$	$B_{5,9}$	$\alpha_{5,7}$	$(0, 1, 0, 0, 0, 1, 0)$	$(0, 1, 1, 1, 0, 1, 0)$
$G_{5,9}$	$G_{5,9}$	$\alpha_{5,8}$	$(0, 1, 0, 0, 0, -1, 0)$	$(0, 0, 0, 0, 1, 0, 0)$
$A_{5,6,9}$	$A_{5,6}$	$\alpha_{5,9}$	$\frac{1}{2}(-1, 1, 1, -1, -1, -1, \sqrt{2})$	$(0, 1, 2, 2, 1, 1, 1)$
$A_{5,7,9}$	$A_{5,7}$	$\alpha_{5,10}$	$\frac{1}{2}(-1, 1, -1, 1, -1, -1, \sqrt{2})$	$(0, 1, 1, 2, 1, 1, 1)$
$A_{5,8,9}$	$A_{5,8}$	$\alpha_{5,11}$	$\frac{1}{2}(-1, 1, -1, -1, 1, -1, \sqrt{2})$	$(0, 1, 1, 1, 1, 1, 1)$
A_5	$A_{5,9}$	$\alpha_{5,12}$	$\frac{1}{2}(-1, 1, -1, -1, -1, 1, \sqrt{2})$	$(0, 1, 1, 1, 0, 1, 1)$
$A_{5,7,8}$	$A_{5,7,8,9}$	$\alpha_{5,13}$	$\frac{1}{2}(-1, 1, -1, 1, 1, 1, \sqrt{2})$	$(0, 1, 1, 2, 1, 2, 1)$
$A_{5,6,8}$	$A_{5,6,8,9}$	$\alpha_{5,14}$	$\frac{1}{2}(-1, 1, 1, -1, 1, 1, \sqrt{2})$	$(0, 1, 2, 2, 1, 2, 1)$
$A_{5,6,7}$	$A_{5,6,7,9}$	$\alpha_{5,15}$	$\frac{1}{2}(-1, 1, 1, 1, -1, 1, \sqrt{2})$	$(0, 1, 2, 3, 1, 2, 1)$
$A_{\mu\nu\rho}$	$A_{5,6,7,8}$	$\alpha_{5,16}$	$\frac{1}{2}(-1, 1, 1, 1, 1, -1, \sqrt{2})$	$(0, 1, 2, 3, 1, 2, 1)$
$B_{4,5}$	$B_{4,5}$	$\alpha_{6,1}$	$(1, 1, 0, 0, 0, 0, 0)$	$(1, 2, 2, 2, 1, 1, 0)$
$G_{4,5}$	$G_{4,5}$	$\alpha_{6,2}$	$(1, -1, 0, 0, 0, 0, 0)$	$(1, 0, 0, 0, 0, 0, 0)$
$B_{4,6}$	$B_{4,6}$	$\alpha_{6,3}$	$(1, 0, 1, 0, 0, 0, 0)$	$(1, 1, 2, 2, 1, 1, 0)$
$G_{4,6}$	$G_{4,6}$	$\alpha_{6,4}$	$(1, 0, -1, 0, 0, 0, 0)$	$(1, 1, 0, 0, 0, 0, 0)$
$B_{4,7}$	$B_{4,7}$	$\alpha_{6,5}$	$(1, 0, 0, 1, 0, 0, 0)$	$(1, 1, 1, 2, 1, 1, 0)$
$G_{4,7}$	$G_{4,7}$	$\alpha_{6,6}$	$(1, 0, 0, -1, 0, 0, 0)$	$(1, 1, 1, 0, 0, 0, 0)$
$B_{4,8}$	$B_{4,8}$	$\alpha_{6,7}$	$(1, 0, 0, 0, 1, 0, 0)$	$(1, 1, 1, 1, 1, 1, 0)$
$G_{4,8}$	$G_{4,8}$	$\alpha_{6,8}$	$(1, 0, 0, 0, -1, 0, 0)$	$(1, 1, 1, 1, 0, 0, 0)$
$B_{4,9}$	$B_{4,9}$	$\alpha_{6,9}$	$(1, 0, 0, 0, 0, 1, 0)$	$(1, 1, 1, 1, 0, 1, 0)$
$G_{4,9}$	$G_{4,9}$	$\alpha_{6,10}$	$(1, 0, 0, 0, 0, -1, 0)$	$(1, 1, 1, 1, 1, 0, 0)$
$B_{\mu\nu}$	$B_{\mu\nu}$	$\alpha_{6,11}$	$(0, 0, 0, 0, 0, 0, \sqrt{2})$	$(1, 2, 3, 4, 2, 3, 2)$
$A_{\mu\nu,9}$	$A_{\mu\nu}$	$\alpha_{6,12}$	$\frac{1}{2}(1, 1, 1, 1, 1, 1, \sqrt{2})$	$(1, 2, 3, 4, 2, 3, 1)$
$A_{4,5,9}$	$A_{4,5}$	$\alpha_{6,13}$	$\frac{1}{2}(1, 1, -1, -1, -1, -1, \sqrt{2})$	$(1, 2, 2, 2, 1, 1, 1)$
$A_{4,6,9}$	$A_{4,6}$	$\alpha_{6,14}$	$\frac{1}{2}(1, -1, 1, -1, -1, -1, \sqrt{2})$	$(1, 1, 2, 2, 1, 1, 1)$
$A_{4,7,9}$	$A_{4,7}$	$\alpha_{6,15}$	$\frac{1}{2}(1, -1, -1, 1, -1, -1, \sqrt{2})$	$(1, 1, 1, 2, 1, 1, 1)$
$A_{4,8,9}$	$A_{4,8}$	$\alpha_{6,16}$	$\frac{1}{2}(1, -1, -1, -1, 1, -1, \sqrt{2})$	$(1, 1, 1, 1, 1, 1, 1)$
A_4	$A_{4,9}$	$\alpha_{6,17}$	$\frac{1}{2}(1, -1, -1, -1, -1, 1, \sqrt{2})$	$(1, 1, 1, 1, 0, 1, 1)$
$A_{4,7,8}$	$A_{4,7,8,9}$	$\alpha_{6,18}$	$\frac{1}{2}(1, -1, -1, 1, 1, 1, \sqrt{2})$	$(1, 1, 1, 2, 1, 2, 1)$
$A_{4,6,8}$	$A_{4,6,8,9}$	$\alpha_{6,19}$	$\frac{1}{2}(1, -1, 1, -1, 1, 1, \sqrt{2})$	$(1, 1, 2, 2, 1, 2, 1)$
$A_{4,6,7}$	$A_{4,6,7,9}$	$\alpha_{6,20}$	$\frac{1}{2}(1, -1, 1, 1, -1, 1, \sqrt{2})$	$(1, 1, 2, 3, 1, 2, 1)$
$A_{\mu\nu,5}$	$A_{4,6,7,8}$	$\alpha_{6,21}$	$\frac{1}{2}(1, -1, 1, 1, 1, -1, \sqrt{2})$	$(1, 1, 2, 3, 2, 2, 1)$
$A_{4,5,8}$	$A_{4,5,8,9}$	$\alpha_{6,22}$	$\frac{1}{2}(1, 1, -1, -1, 1, 1, \sqrt{2})$	$(1, 2, 2, 2, 1, 2, 1)$
$A_{4,5,7}$	$A_{4,5,7,9}$	$\alpha_{6,23}$	$\frac{1}{2}(1, 1, -1, 1, -1, 1, \sqrt{2})$	$(1, 2, 2, 3, 1, 2, 1)$
$A_{\mu\nu,6}$	$A_{4,5,7,8}$	$\alpha_{6,24}$	$\frac{1}{2}(1, 1, -1, 1, 1, -1, \sqrt{2})$	$(1, 2, 2, 3, 2, 2, 1)$
$A_{4,5,6}$	$A_{4,5,6,9}$	$\alpha_{6,25}$	$\frac{1}{2}(1, 1, 1, -1, -1, 1, \sqrt{2})$	$(1, 2, 3, 3, 1, 2, 1)$
$A_{\mu\nu,7}$	$A_{4,5,6,8}$	$\alpha_{6,26}$	$\frac{1}{2}(1, 1, 1, -1, 1, -1, \sqrt{2})$	$(1, 2, 3, 3, 2, 2, 1)$
$A_{\mu\nu,8}$	$A_{4,5,6,7}$	$\alpha_{6,27}$	$\frac{1}{2}(1, 1, 1, 1, -1, -1, \sqrt{2})$	$(1, 2, 3, 4, 2, 2, 1)$

Table 2: The correspondence between the positive roots $\alpha_{m,n}$ of the U -duality algebra $\mathcal{E}_{7(7)}$ and the scalar fields parameterizing the moduli space for either IIA and IIB compactifications. The notation $\alpha_{m,n}$ ($m = 1, \dots, 6$, $n = 1, \dots, d(m)$) for the positive roots was introduced in [19]. The seven Cartan generators correspond to the dilaton and the six radii.